

# The stopping power of heavy ions in liquid water at low velocity $v \leq v_0 Z^{2/3}$

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**Abstract-** In present work, a stopping power of heavy ions like C, N and O in liquid water H<sub>2</sub>O has been studied at low velocity regime,  $v \leq v_0 Z_1^{2/3}$  using Lindhard and Bohr formula for electronic interaction. Bohr Theory has been extended to work even at low velocity limit. The effective charge is taking in the consideration. Good agreement achieved with the output of SRIM software result.

**Keywords-** Stopping power of heavy ion, heavy ion, Bohr theory, Liquid water, low-velocity stopping.

## I. INTRODUCTION

The stopping of ions penetrating through matter is governed by a number of scattering processes such as (i) excitation and ionization of target electron, (ii) electron capture, (iii) projectile excitation and ionization, and (iv) recoiling nuclei. While this classification is not unique and individual processes are not necessarily uncoupled, existing theory [1-2]. The central role of the projectile charge as well as exchange in conjunction with stopping phenomena was discussed by Bohr and Lindhard [3]. It has been common to derive up heavy ion stopping into three regimes a low velocity regime where the stopping force is taken to be proportional to the projectile speed  $v$  and given roughly by the Lindhard [4], a high velocity regime characterized by the Bethe formula [5], and the intermediate regime around and above the stopping maximum which has most often been characterized by the Bethe-type formula in conjunction with some effect ion charge in a classic paper “on the theory of the decrease of velocity of moving electrified particles on passing through matter”. Bohr [6] derived an expression for the stopping power cross section per target electron (charge – e, mass m) of a material which is given as;

$$S = \frac{2\pi Z_1^2 e^4}{mv^2} \ln \left[ \frac{Cmv^3}{Z_1 e^2 \omega} \right] \quad (1)$$

Where:  $C=1.1229$ ,  $Z_1$  atomic number of incident charge particle,  $\omega$  classical resonance frequency,  $v$  the velocity of incident particle

Bloch's extension of Bethe's theory [7]. Contain the two results as limiting case. Bloch's formula reduces to Bethe's for projectiles of low charge, especially protons and deuterons while Bohr's result is approached in the limit of

$$k = \frac{2z_1 v_0}{v} > 1 \quad (2)$$

$v_0 = e^2/\hbar$  being the Bohr velocity [6]. Liquid water was taken into consideration because it is the only common substance found naturally in all three common states of matter and it is essential for all life on Earth. Water usually makes up 55% to 78% of the human body [8].

By using SRIM program one can calculated the stopping power of heavy ions and comparing the result from Bohr formula, Lindhard, and experimental data both this data from SRIM [9].

## II. THEORY

There are many ways to calculate the stopping power at low velocity regime, including Lindhard and Bohr equation's in this research methods to present. Lindhard equations by the following equation [10]:

$$S_s = 4\pi Z_1^2 Z_2 e^4 \frac{Z_1^{7/8} Z_2}{(Z_1^{2/8} + Z_2^{2/8})^{2/8}} \left( \frac{v}{v_0} \right) \quad (3)$$

As well as the effective charge for heavy ions carry typically a number of electrons with increasing penetration depth the ionic charge  $Z^*$  fluctuates around a certain equilibrium value, which is the commonly estimated from an expression of the type of [11,12]

$$Z^* = Z_1(1 - e^{-v/v_{th}}) \quad (4)$$

Where  $v_{th} = Z_1^{2/3} v_0$  is the Tomas-Fermi velocity, interaction between a bare projectile and target electrons in Bohr Theory are classified into close and distant collisions according to impact parameters below or above a certain critical value  $p_0$  [13]. Close collisions are treated as free-coulomb interactions, resulting in the following dependence of the energy transfer  $T$  versus impact parameter  $p$ , [14]:

$$T_{close}(p, v) = \frac{2Z_1^2 e^4}{mv^2 p^2} \frac{1}{1 + (b/2P)^2} \quad (5)$$

When  $b = 2Z_1 e^2 / mv^2$  is the collision diameter. This relation follows directly from Rutherford's law ( $\tan \theta/2 = b/2p$ ) and,  $T = 2mv^2 \sin^2 \theta/2$ ,  $\theta$  denoting the center-of-mass scattering angle. Binding of target is taken into account in distant interactions and enters through a classical resonance frequency  $w$ ; the electric field of the projectile is taken in the dipole limits, i.e., its spatial variation across the diameter of the atom is ignored. This yields [14]:

$$T_{dist}(p, v) = \frac{2Z_1^2 e^4}{mv^2 p^2} \left\{ \left[ K_0 \left( \frac{\omega p}{v} \right) \right]^2 + \left[ K_1 \left( \frac{\omega p}{v} \right) \right]^2 \right\} \left( \frac{\omega p}{v} \right)^2$$

Where  $K_0$  and  $K_1$  are modified Bessel functions in standard notation [15]. So one can refer to stopping power by:

$$\begin{aligned} S &= 2\pi Z_2 \int_0^\infty T(p) p dp \\ &= \int_0^{p_0} 2\pi p dp T_{close}(p) + \int_{p_0}^\infty 2\pi p dp T_{dist}(p) \end{aligned} \quad (7)$$

Where,  $p_0$  is a critical impact parameter separating close from distant interactions. Newton Raphson [16], method has been used to find  $P_0$ . The stopping cross section in terms of close and distant collisions is given by

$$S(P) = S_{close}(P) + S_{dist}(P) \quad (8)$$

Inserting equations 1, 2 in eq. 4 leading to [17]

$$S_{close}(p) = \frac{2\pi Z_1^2 e^4}{mv^2} \ln \left[ 1 + \left( \frac{2p_0}{b} \right)^2 \right] \quad (9)$$

And

$$S_{dist}(p) = \frac{4\pi Z_1^2 e^4}{mv^2} \left( \frac{\omega p_0}{v} \right) K_0 \left( \frac{\omega p_0}{v} \right) K_1 \left( \frac{\omega p_0}{v} \right) \quad (10)$$

Where  $S_{dist} = S_{close}$  at  $p = p_0$  also when  $b \ll P_0 \ll v/\omega$  then one get,

$$p_0 = \left( \frac{Cv^2}{\xi \omega^2} \right)^{1/2} \quad (11)$$

A program has been written in fortran-90 for present numerical work [20].

### III. RESULTS AND DISCUSSION

The stopping cross-section given by Lindhard is directly proportional with projectile velocity; While Bohr Eq. (7) is inversely with,  $v$ . And directly with Bohr Parameter,  $\log \left( \frac{1.1223 mv^3}{Z_1 e^2 w} \right)$ . The electronic stopping cross-section,  $S_e$  in ( $eV \cdot cm^2 / atom$ ),  $10^{-15}$  for C, N and O ions in liquid water at different velocity  $v \leq v_0 Z_1^{2/3}$  in shown in fig. (1). There is large different between the result of Lindhard, Bohr and SRIM software [18], at low velocity  $v \leq v_0 Z_1^{2/3}$  as shown in fig. 1. This is due to the effective charge parameter. The agreement is more better when include the effects of effective charge given in Eq. (4) as shown in Fig. 2. To get more reliable result one must add nuclear stopping cross-section,  $S_n$  to the electronic stopping cross-section,  $S_e$ .

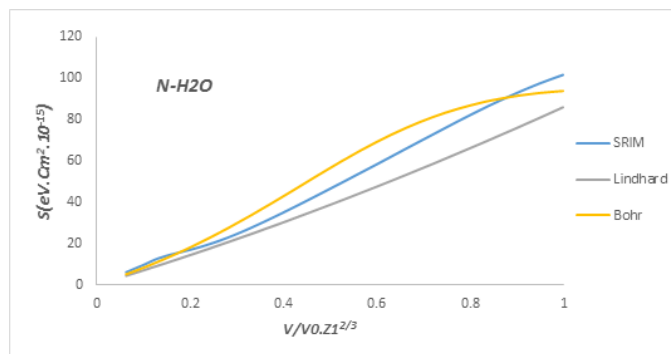
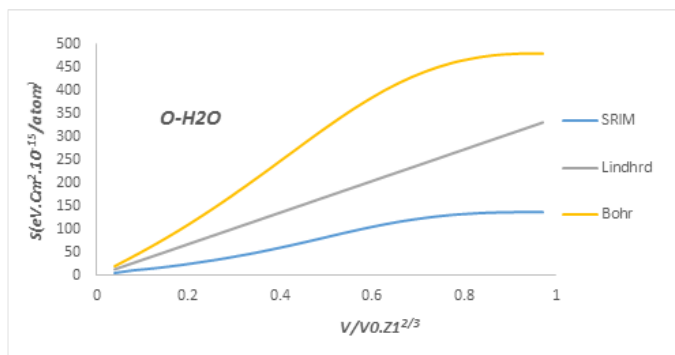
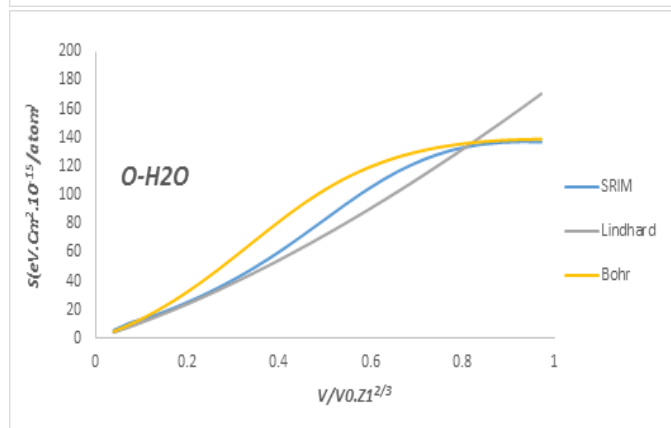
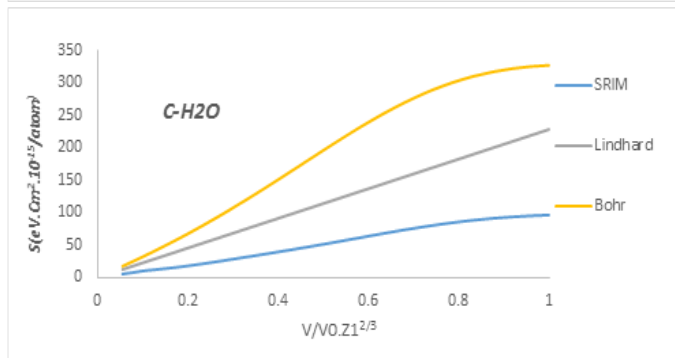
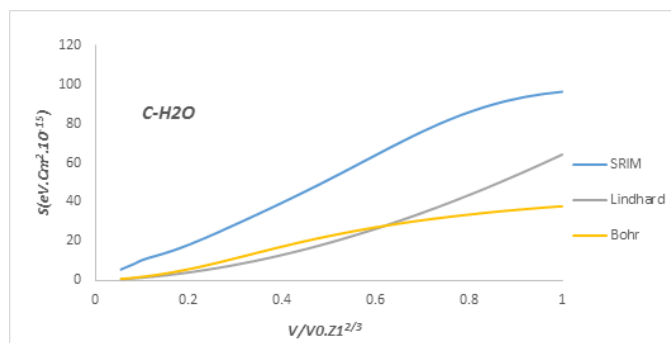
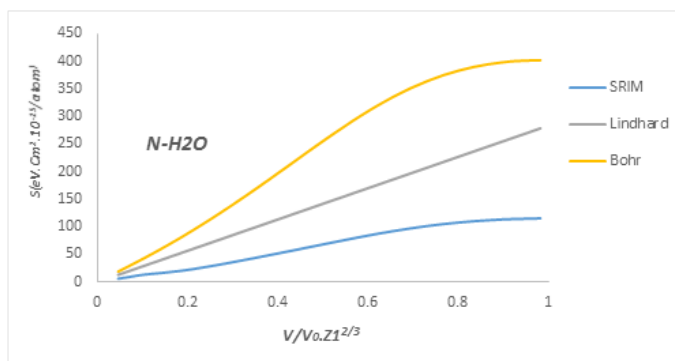


Fig. 1 The stopping cross-section  $S$  in  $eV.Cm^2.10^{-15}/atom$  for O, C and N in liquid water, H<sub>2</sub>O, SRIM [18], Lindhard and Bohr at difference velocity  $v/v_0.Z_1^{2/3}$

Fig. 2 Same as Fig.1 except taking in the consideration the effective charge.

#### IV. CONCLUSION

- i. For stopping power calculated with heavy ions at low energy, it is so imperative to take in consideration correct for the nuclear contribution
- ii. In the low velocity, it is very important to take the effective charge in consideration.
- iii. One must include the effect of Curvature variation of stopping power at low velocity region  $v \leq v_0 Z_1^{2/3}$  as mentioned in [19].
- iv. The extensions limit to Bohr theory as given in Eq. (1) is important at low velocity regions  $v \leq v_0 Z_1^{2/3}$ .

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